CS 224 - Radiometry in "Flatland"

Released: 2/1/19
Due: 2/8/19 by 11:59pm EST
Submission instructions: Run `cs224_handin` from a directory containing your solutions. This can be in any human-readable format: a LaTeX'ed PDF, a scanned document, etc.

Everything we've said about the rendering equation makes sense in 2D, which is to say, you can imagine things in "Flatland" that correspond to surfaces in ordinary space, namely line segments or arcs. And these can then emit or receive light. And from that, you can develop a notion of "radiance," and a notion of the BRDF or BSDF. We'll refer to the radiance notion here as *fradiance* (for "flatland radiance"), and similarly prepend "f" to various other terms.

1. What should be units of fradiance be? (Assume that photons in flatland still carry energy measured in Joules.)

2. In Lecture 3 (Rendering Equation and Monte Carlo Integration), we saw that one of the properties of the BRDF is that it must conserve energy. That is, for all possible reflected light directions ω_r, it must be the case that $\int_{\mathcal{H}^2} f_r(\omega_i \to \omega_r) \cos\theta_i \mathrm{d}\omega_i \leq 1$. This means that for a constant BRDF $f_r(\omega_i \to \omega_r) = f_r$, $f_r \leq \frac{1}{\pi}$. That is, the BRDF is normalized by a factor of π. What is the normalization factor for a constant fBRDF?

3. Write the rendering equation for this 2D world. In your answer, you may wish to use the following functions:

   - $tr(x, \omega)$, the *transport function*, returns the first point x' hit by the ray starting from x and traveling in direction ω.
   - $\omega(x, \theta)$ returns the direction ω which is at an angle θ from the surface normal at point x.
   - $\theta(x, \omega)$ is the inverse of $\omega(x, \theta)$.

4. Imagine a scene consisting of a totally diffusely reflective floor and a radiant but totally absorbtive ceiling, both **infinite** (the "floor" is the x-axis; the ceiling is the line at $y = 2$). The ceiling is emitting light downward, uniformly in all directions, and the fradiance along each ray is 1 (Q: "One what?" A: "1 unit, where the units are the answer to question 1"). What's the radiosity at any point on the ceiling? Remember that in three-space, radiance is defined as power per unit solid angle per unit projected area; the corresponding definition for fradiance will be useful here.

5. In the scene from the previous question, what's the irradiance at any point on the floor? How does this relate to the radiosity at any point on the ceiling (i.e. your answer to the last question)? Why must this relationship hold?

6. Take the same scene as in the previous question, but now the floor is perfectly (i.e. mirror) reflective. What's the irradiance at any point on the ceiling?

7. Take the same scene as in the previous question, but now the ceiling is also 50% diffuse reflective. Now what's the irradiance at any point on the floor?